N88-23108

Evaluation of a Theory for Pressure-Strain Rate

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A theoretical expression for the slow part (the non-linear fluctuation part) of the pressure-strain rate is compared with simulations of anisotropic homogeneous flows. The objective is to determine the quantitative accuracy of the theory and to test its prediction that the generalized Rotta coefficient, a non-dimensionalized ratio of slow term to the Reynolds stress anisotropy, varies with direction and can be negative. Comparisons are made between theoretical and simulation values of the slow term itself and of the generalized Rotta coefficients. The implications of the comparison for two-point closure theories and for Reynolds stress modeling are pointed out.

1. Introduction and background

The slow pressure-strain rate correlation Φ^s_{ij} is a key term that occurs in Reynolds stress modeling. Until recent years it was almost universally modeled according to Rotta's (1951) prescription as

$$\Phi^s_{ij} = -C\frac{\epsilon}{q^2} \quad b_{ij}, \qquad \text{(empirical model)}$$

where $b_{ij} =< u'_i u'_j > -[1/3(q^2\delta_{ij})]$, u'_i is the fluctuating velocity along direction i, $q^2 =< u_i u_i >$ is twice the turbulent kinetic energy density, ϵ is the rate of turbulent kinetic energy dissipation, and C is an empirical constant referred to as Rotta's constant. However, Lumley (1978) has shown that C cannot be constant and more recently, it has been shown (Weinstock, 1981; 1982; 1985) that C is neither constant nor the same for different directional components ij. These variations occur because $\Phi^{(s)}_{ij}$ depends on more than one scale of the turbulence field, and, in addition, the scales vary with direction.

One way to account for the effect of all scales is for the slow term to be derived by a two-point closure theory. Such a derivation has been carried out (Weinstock, 1981; 1982; 1985), a principal result of which was that Φ^s_{ij} can be expressed as an integral over scalar energy spectra $E_{ij}(k)$. Standard closures such as the DIA (Kraichnan, 1959) and EDQNM (e.g., Cambon et al., 1981; Bertoglio, these proceedings) are much more ambitious, and correspondingly complex, since they determine the energy spectrum itself. Here, $E_{ij}(k)$ is taken from simulations. It is hoped that the relative simplicity will allow the present theory to be applied to a wider class of flows – including those with relatively large anisotropies after suitable modeling of

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the scalar spectra. This is encouraged by a "universal" character found for the theoretical Φ^s_{ij} in which its dimensionless (Rotta) coefficients are very insensitive to the shape of the energy spectrum in the small k (energy producing) region, provided that the Reynolds number is not too small. We believe that such a universality is crucial for predictive modeling of flows.

The theoretical Φ^s_{ij} to be tested is given by

$$\Phi^s_{ij} = -C_{ij}\epsilon b_{ij}, \qquad \text{(No sum on } i \text{ and } j) \tag{1a}$$

$$C_{\alpha\alpha} = 1.08\left(\frac{2\pi}{3}\right)^{\frac{1}{2}}\left(\frac{1}{\epsilon k_o^{1/3} q b_{\alpha\alpha}}\right)\int_0^\infty dk_1 \int_0^\infty dk_2$$

$$\times \quad \frac{k_1^2 k_2^2 E(k_2)[E_{\alpha\alpha}(k_1) - \frac{1}{3}E(k_1)]}{(k_1^2+k_2^2)^{4/3}} H(k_1,k_2), \tag{1b}$$

$$E(k) = \frac{1}{2} \quad [E_{11}(k) + E_{22}(k) + E_{33}(k)],$$

$$H(k_1,k_2) \approx \left[2 - \frac{2.4k_2^2}{k_1^2+k_2^2} - 0.08\left(\frac{4k_1k_2}{k_1^2+k_2^2} - 1\right) + \frac{2}{3}\left(1 - \frac{2k_1^2}{k_1^2+k_2^2}\right)\right],$$

where the C_{ij} are dimensionless coefficients referred to as generalized Rotta coefficients, $E_{\alpha\alpha}(k)$ is the scalar spectrum for kinetic energy along direction α, $k_o = (3\beta)^{3/2}\epsilon/q^3$, and β is the Kolmogorov constant. The various numerical factors arise from angular integrations of spectra in wave space. The specific angular dependence of the spectra had to be modeled to make this possible. The off-diagonal elements C_{12}, C_{13}, C_{23} are given elsewhere (Weinstock, 1981) and have not been evaluated. A much simpler form of $C_{\alpha\alpha}$ for use in Reynolds stress modeling is obtained by use of the model spectrum $E_{\alpha\alpha} = \beta\epsilon_{\alpha\alpha}^{2/3}k^{-5/3}$ for $k_o \leq k \leq k_\nu$, and $E_{\alpha\alpha} = \beta\epsilon_{11}^{2/3}k^m k_o^{-m-5/3}$ for $k < k_o$, where k_ν is the viscous "cut-off" wavenumber. We refer to this $E_{\alpha\alpha}$ as the model spectrum.

Our primary goal is to test expression (1) by comparison with computer simulations. This test has also implications for standard two-point closures in general, since such closures have in common with our closure the neglect of a two-time fourth-order velocity cumulant.

Our article is outlined as follows: Straight forward comparisons are given in Sec. 2 where values of $\Phi^s_{\alpha\alpha}$ and $C_{\alpha\alpha} = -\Phi^s_{\alpha\alpha}/(\epsilon b_{\alpha\alpha})$ obtained directly from simulations are compared with (1b). Improvements and generalizations of the theory suggested by simulations are in Sec. 3, and Sec. 4 contains suggestions for further simulation tests.

2. Comparison between theory and simulation

The theory was compared with several simulations of homogeneous shear and straining flows. Typical examples are given in Figures 1 through 3 for two cases of homogeneous shear ($S = U_{1,2}$, simulation runs C128U and C128X of Rogers, Moin

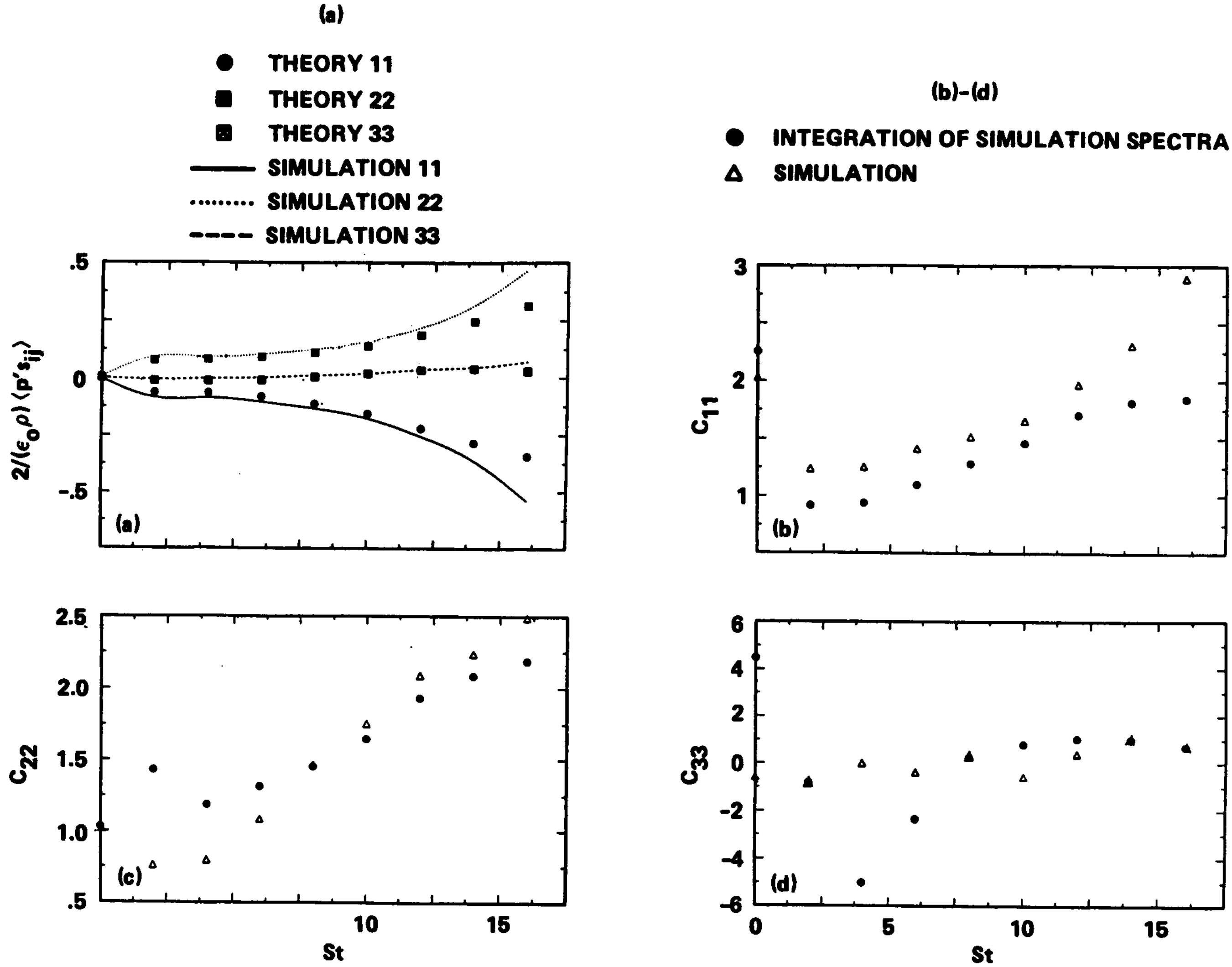

FIGURE 1. Slow pressure strain components for homogeneous shear (moderate shear case C128U of Rogers, Moin & Reynolds, 1986). (a) Comparison of theory with simulation for the diagonal components. (b)-(d) Generalized Rotta coefficients as computed from the simulation data and using the simulation spectra in the theory.

and Reynolds 1986) and for plane strain (strain directions are 2 and 3, simulation run PXA of Lee and Reynolds 1985). For the shear cases the horizontal axis is the total shear, St. For the plane strain case the horizontal axis is the eddy turnover time. Figures 1 show the evolution of $\Phi^s_{\alpha\alpha}$ scaled on the initial dissipation rate, and $C_{\alpha\alpha}$ for run C128U, Figures 2 for run C128X and Figures 3 for plane strain run PXA. Each graph includes simulation and theoretical values.

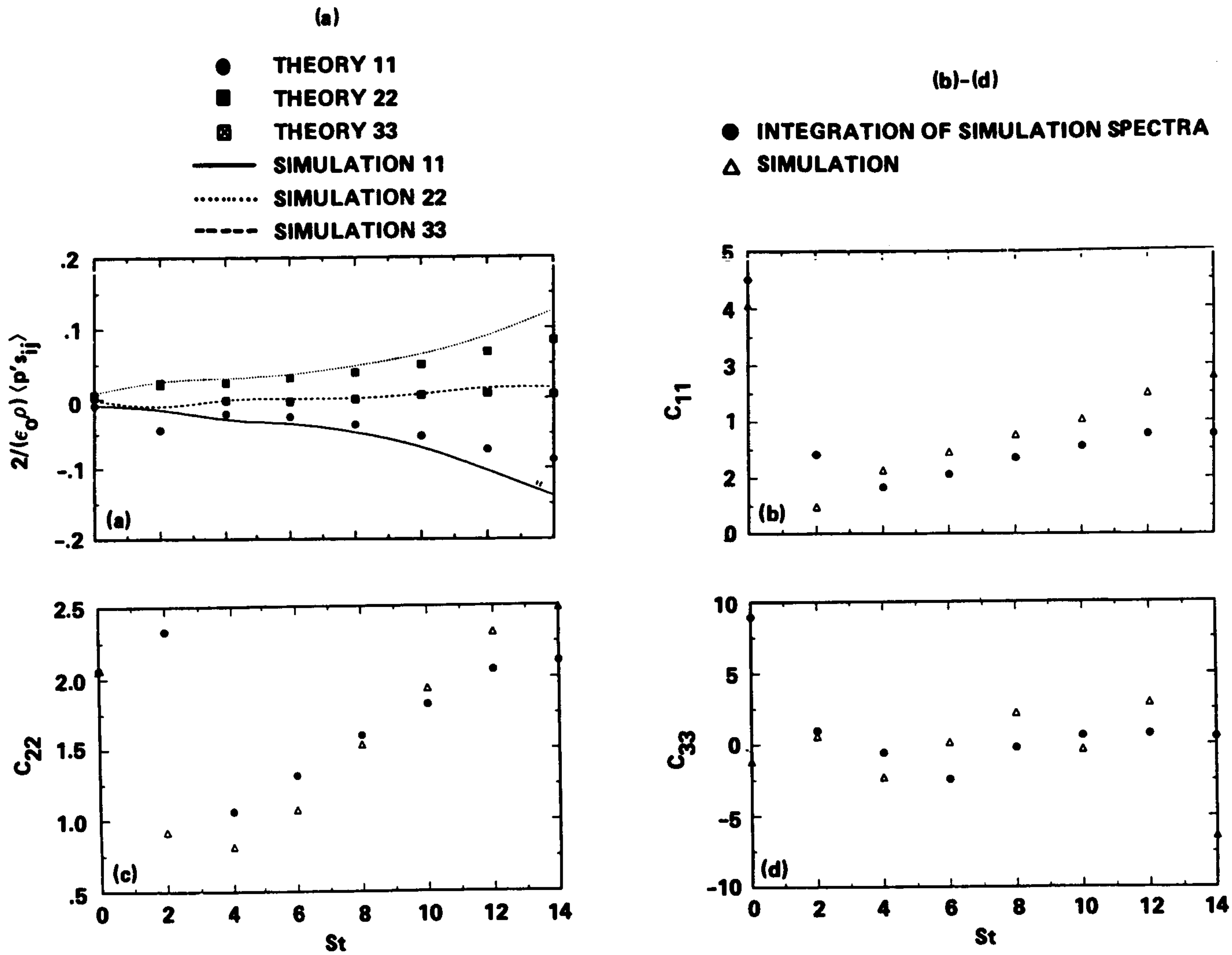

FIGURE 2. Slow pressure strain components for homogeneous shear (high shear case C128X of Rogers, Moin & Reynolds, 1986). a) Comparison of theory with simulation for the diagonal components. b)-d) Generalized Rotta coefficients as computed from the simulation data and using the simulation spectra in the theory.

(a) Dominant features of the (slow) pressure-strain rate

The simulation data show the following features:

- $C_{\alpha\alpha}$ varies between components.
- The normal components C_{11}, C_{22}, C_{33} each vary greatly during the simulations. For example, C_{11}, in plane strain run PXA, varies from -5 To +10 (in the unstrained direction). The Reynolds number defined as $q^4/(\nu\epsilon)$ varied between 39.1 nad 69. Another example is that C_{22} varies from 0.6 to 2.5 in homogeneous shear flow (C128U).
- C_{11} can be negative for many conditions. However, this does not imply that

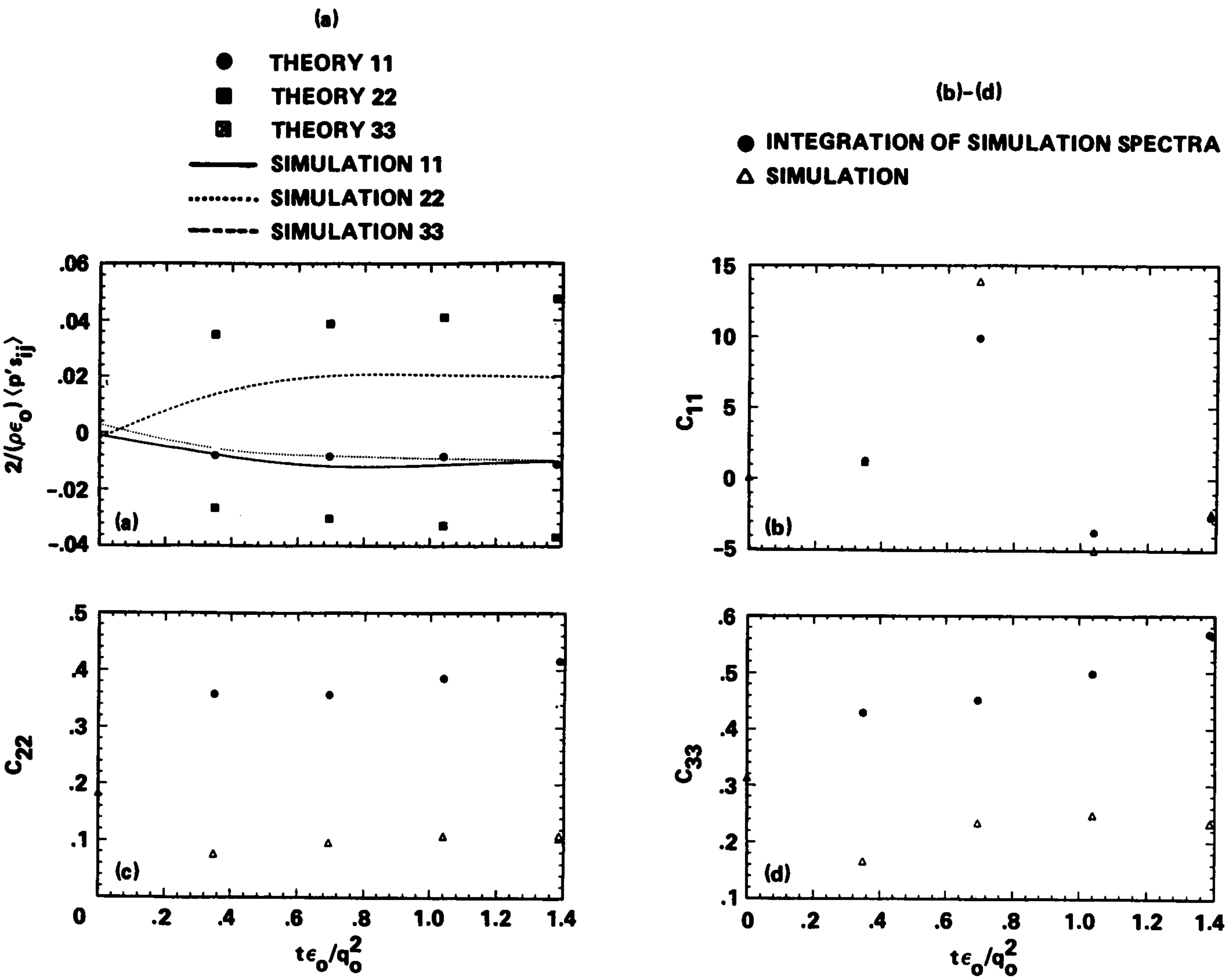

FIGURE 3. Slow pressure strain components for homogeneous plane strain (case PXA of Lee & Reynolds, 1985). a) Comparison of theory with simulation for the diagonal components. b)-d) Generalized Rotta coefficients as computed from the simulation data and using the simulation spectra in the theory.

the flow will not return to isotropy were the mean deformation to be removed at that instant. The Lumley return to isotropy tensor, of which the pressure strain rate is only a part, determines this.

Each of these qualitative features is predicted by the theory. There is good quantitative agreement of the pressure strain rate for the shear cases, discounting times larger than $St = 12$ where the "box" size has an important influence. The agreement is weaker for the case of plane strain. In the comparisons for the generalized Rotta coefficients $C_{\alpha\alpha}$, the theoretical $C_{\alpha\alpha}(\alpha = 1, 2, \text{or } 3)$ generally follows the trend of the simulations; being small whenever the simulation $C_{\alpha\alpha}$ is small and large when-

ever the $C_{\alpha\alpha}$ is large, and most notably, passing through zero at the same time that the simulation $C_{\alpha\alpha}$ (see Figure 3b) does.

During straining flows, the values of $C_{\alpha\alpha}$ in the strained directions, were always overpredicted by the theory. This discrepancy might be accounted for by the strong temporal variations of $b_{\alpha\alpha}$ which violates the present assumptions in the theory which limit it to slow variations of $b_{\alpha\alpha}$ and low mean strain-rate. Indeed, when estimates are made to account for time-scale variation of $b_{\alpha\alpha}$ by calculating the change in the Lagrangian time-scale that is used to derive (1b), the discrepancies are reduced. However, for modeling purposes the original, unextended theory may be sufficient.

(b) Unexpected features of $\Phi^s_{\alpha\alpha}$

- Simulations show that $C_{\alpha\alpha}$ can be very small (much less than unity) in straining flows for a wide range of anisotropies. This smallness was unexpected, although it is contained in the theory. Small $C_{\alpha\alpha}$ implies that intercomponent energy transfer is a very weak process in straining flows.
- $C_{\alpha\alpha}$ can vary significantly with strong temporal variations of kinetic energy.
- Surprising is the extent of difference between C_{11} and C_{33} in homogeneous shear flows (Figures 1b and 1d).

These features are also found in the theory with small quantitative discrepancies.

3. Improvement and generalization of the theory

(i) An intriguing proposition is to derive $\Phi_{ij}(\mathbf{x}) = < p^s(\mathbf{x}')S_{ij}(\mathbf{x}'+\mathbf{x}) >$, the two-point correlation of the slow pressure-strain, from the theory. This was suggested by Brasseur and obtained from simulations by Brasseur and Lee, Schiestel and Rogallo (these proceedings). This correlation provides a more severe and detailed test of closure theory than does $C_{\alpha\alpha}$. It also provides a direct link between spatial structures and Reynolds stress modeling. The theoretical $\Phi_{ij}(\mathbf{x})$ was worked out during the summer school, but was not compared with simulations at the time; for example, one component of $\Phi^s_{ij}(\mathbf{x})$ is

$$\Phi^s_{22}(x_1) = -\left(\frac{2\pi}{3}\right)^{\frac{1}{2}} \frac{1}{k_o^{1/3} q} \int_0^\infty dk \int_0^\infty dk_a \int d\theta \sin\theta \cos(kx_1\cos\theta)$$

$$\times \frac{k^2 k_a^2 E(k_b) H}{(k^2+k_a^2)^{4/3}} \left[\sin^2\theta E_{22}(k_a) - \frac{5}{2}\sin^2\theta \quad \cos^2\theta \quad E_{11}(k_a) - \frac{5}{8}\sin^4\theta \quad E_{33}(k_a)\right]$$

where

$$k_b^2 = (k^2+k_a^2)\left[1+\frac{4}{3}\frac{k^2k_a^2}{(k^2+k_a^2)^2}\right]$$

(ii) Extend the theory to include spatial and temporal variations.

(iii) Revise the model scalar spectra $E_{\alpha\alpha}(k)$ to account for small Reynolds number.

As pointed out by Bertoglio the theory does not account for deformation of wave vectors by the mean strain rate and should be modified accordingly.

(iv) Attempt to model the rapid pressure-strain rate, by a k-space closure.

(v) Compare the theoretical pressure-strain rate with a $k-$ space model of Schiestel (these proceedings).

4. Suggestions for future simulations

(a) Regarding the pressure-strain rate:

(i) Compare the theoretical two-point pressure-strain $< p^s(\mathbf{x}')S_{ij}(\mathbf{x}'+\mathbf{x}) >$ with simulation.

(ii) Generalize the theoretical $< p^s S_{ij} >$ to apply to channel flow, and then test with simulations.

(iii) Using simulations, calculate the two-time fourth-moment velocity cumulant. Determine the time scale for its decay. In particular, determine if this time scale is shorter than the time scale for decay of second-moment correlations. Such an ordering of time scales is basic for k-space closure theories in general, and also for the present theory.

(b) Regarding a theory for modeling inhomogeneous flows:

(i) Compare simulation values of $< u_i'u_j'u_k' >$, $< \theta u_i'u_j' >$, $< \theta^2 u' >$, $< u_i'u_j'\partial p/\partial x_k >$, $< \theta u_i'\partial p/\partial x_j >$ with the eddy-damped quasi-Gaussian approximation (EDQG) and with a recent theory. These quantities are basic to modeling weakly inhomogeneous and stratified flows.

(ii) Verify whether or not the cumulant of $< (\mathbf{u}' \cdot \nabla \mathbf{u}')_i u_i' \mathbf{u}_i' > = < u'^2 > \ \nabla \ < \mathbf{u}_i'^2 >$. This was derived by a theory and, if true, contradicts the quasi-Gaussian assumption for inhomogeneous flows.

acknowledgment

We are grateful to M. J. Lee, N. N. Mansour and R. S. Rogallo for invaluable assistance with the databases, and for their interest despite their involvement in several other CTR projects.

REFERENCES

CAMBON, C., JAENDEL, D. & MATHIEU, J. 1981 . *J. Fluid Mech.* **104,** 247–262.

KRAICHNAN, R. 1959 . *J. Fluid Mech.* **5,** 497–543.

LEE, M. J. & REYNOLDS, W. C. 1985 Numerical experiments on the structure of homogeneous turbulence. *Report No. TF-24.* Mech. Engrg., Stanford Univ.

LUMLEY, J. L. 1978 . *Adv. Appl. Mech.* **18,** 123–176.

ROGERS, M. M., MOIN, P., & REYNOLDS, W. C. 1986 The structure and modeling of the hydrodynamic and passive scalar fields in homogeneous turbulent shear flow. *Report No. TF-25.* Mech. Engrg., Stanford Univ.

ROTTA, J. 1951 . *Z. Phys.* **129,** 547-572.

WEINSTOCK, J. 1981 . *J. Fluid Mech.* **105,** 369–396.

WEINSTOCK, J. 1982 . *J. Fluid Mech.* **116,** 1-29.

WEINSTOCK, J. 1985 . *J. Fluid Mech.* **154,** 429–447.